TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 238

PROPELLER DESIGN

A SIMPLE METHOD FOR DETERMINING THE STRENGTH OF PROPELLERS - IV

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Washington
June, 1926

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Summary

The object of this report, the last of a series of four on propeller design, is to describe a simple method for determining whether the strength of a propeller of a standard form is sufficient for safe operation. An approximate method of stress analysis is also given.

Introduction

After a propeller has been designed to meet certain aerodynamic requirements it is often desired to know whether it has sufficient strength for safe operation. The accurate stress analysis of a propeller is a complex and laborious process, and the approximate methods used in routine design work are necessarily inaccurate.

In this report a simple relation is given for determining whether any propeller of standard Navy form is sufficiently strong for its particular operating requirements. The method used can be adapted to any family of propellers of one general form.

Approximate Stress Analysis

Three main types of forces act on propeller blades:

1. Centrifugal forces,
2. Aerodynamic forces,
3. Gyroscopic forces.

The actual stresses are of a very complicated nature, but in routine design work on standard propellers only the two most important need be considered:

1. Uniform tensile stress due to centrifugal forces.
2. Maximum tensile stress caused by bending due to the lift of the airfoil.

The stresses due to centrifugal force will be considered first. The centrifugal force acting on any given particle in the propeller is represented by the equation $C.F. = \frac{W}{g}\,\omega^2 r$, where W is the weight of the particle, g is the acceleration due to gravity, ω is the angular velocity in radians per second, and r is the distance of the particle from the center of rotation. If N equals the revolutions per minute, the angular velocity ω becomes $\frac{2\,\pi\,N}{60}$ and the equation takes the form of $C.F. = \frac{Wr}{g}\left(\frac{2\,\pi\,N}{60}\right)^2$.

The propeller is divided into stations usually spaced either 6 in. or 15 per cent R (tip radius) apart along the radius. Considering sections 1 in. in radial length at each station, the weight W, of each section becomes $W = Aw$, where A is the cross sectional area of the section in square inches and w is the density of the material in pounds per cubic inch. The centrifugal force of a section is given by

$$\frac{d\,(C.F.)}{dr} = \frac{W}{g}\,\omega^2\,r = A r\,\frac{w}{g}\left(\frac{2\,\pi\,N}{60}\right)^2$$

and the total centrifugal force at any station will be

$$C.F. = \int_r^R A r\,\frac{w}{g}\left(\frac{2\,\pi\,N}{60}\right)^2 dr.$$

In practice this integration is performed graphically. The centrifugal force $\frac{d(C.F.)}{dr}$ is calculated for each section and plotted against the radius of the station, and a curve is drawn through the points (Fig. 1). Then the total centrifugal force at each station will be represented by the area under the curve from the tip radius of the propeller to the radius of the station. The tensile stress in each section is obtained by dividing the total centrifugal force for the station by the area of the section, or $S_t = \frac{C.F.}{A}$.

The above computations for all of the stations of a propeller are conveniently arranged in Table I. The stations, blade widths b, and the upper and lower cambers h_U and h_L, are obtained from the propeller layout or design. The areas of the various sections are then computed by means of the formula $A = 0.74\ b\ h$, where h is the total maximum camber of the section. This is an approximate formula which is correct with practical limits for the R.A.F. 6 (modified) airfoil section used in propeller work.

The following is a calculation of the tensile stress due to C.F. for the 3-foot station of the propeller given in Table I.

$$N = 1750$$

$$r = 36 \text{ in.} = 3 \text{ ft.}$$

$$b = 7.65 \text{ in.}$$

$$h_U = .57$$

$$h_L = 0$$

1. $A = .74\ b\ h$

$$= .74 \times 7.65\ (.57 + 0)$$

$$= 3.23 \text{ sq.in.}$$

2. $Ar = 3.23 \times 36 = 116.4 \text{ in.}^3$

3. $\dfrac{d\ (C.F.)}{dr} = Ar\ \dfrac{w}{g}\left(\dfrac{2\ \pi\ N}{60}\right)^2 = Ar \times \dfrac{.103}{32.2}\left(\dfrac{2\pi \times 1750}{60}\right)^2$

$$= 108\ Ar = 108 \times 116.4 = 12,580 \text{ lb. per in. radius.}$$

4. $S_t = \dfrac{C.F.}{A} = \dfrac{13,380}{3.23}$ (planimetered) $= 4140$ lb./sq.in.

In bending, the propeller acts as a cantilever beam. The load on the beam is taken as the lift of the airfoil only, no other aerodynamic forces being considered. The load, then, is proportional to $\rho\ C_L\ AV^2$, where ρ is the density of the air in mass units (which may be taken as .00237), C_L is the absolute lift coefficient of the airfoil. A is the airfoil area, and V is the velocity relative to the air. The blade is divided into 1-foot units.

The lift per foot of radius is given by $L = \frac{\rho C_L b (V_r)^2}{2}$, where b is the blade width in feet, and V_r is the velocity of the blade section with respect to the air in feet per second, or $(V_r)^2 = (V')^2 + \left(\frac{2 \pi rN}{60}\right)^2$. V' is the velocity of the plane in feet per second plus an empirical inflow factor, or $V' = V (1 + \frac{1}{2} S)$, where S is the slip.

The lift per unit radius L, is calculated for each station and plotted against the radius of the station in the same manner as for centrifugal force. This gives a load curve for the propeller as shown in Fig. 2. The shear at any point is given by the equation $F_s = \int_r^R L\, dr$, which is the area under the load curve and is found graphically. The values of F_s at each station are then plotted in like manner, giving a shear curve (Fig. 3). The bending moment at any point is $M = \int_r^R S\, dr$, which is the area under the shear curve. The maximum tensile stress S_t at each station is then calculated by means of the relation $S_t = \frac{My_t}{I}$, where I is the least moment of inertia of the section and y_t is the distance from the neutral axis to the outermost fiber on the working face. The neutral axis for the least moment of inertia runs through the c.g. of the section and is assumed parallel to the chord of the airfoil. This is not strictly true but leads to very slight errors. The positions of the c.g.'s of single and double cambered R.A.F.6 (modified) airfoils are shown in Fig. 6.

The least moment of inertia I is given approximately by $I = .0472\ b\ h^3$ for a single cambered section, and

$$I = .0472\ b\ (h^3_U + h^3_L) + .112\ b\ h_U\ h_L\ (h_U + h_L)$$

for a double cambered section..

The calculations of S_t due to bending are shown for all of the stations of the propeller in the second part of Table I. For the 3-foot station the calculations in detail are as follows:

1. $y_t = .416\ h = .416 \times .57 = .237$ in.
2. $I = .0472\ b\ h^3 = .0472 \times 7.65 \times .57^3 = .067$ in.^{4}
3. $\dfrac{I}{y_t} = \dfrac{.067}{.237} = .283$ in.^{3}
4. $\dfrac{2\pi rN}{60} = \dfrac{2\pi \times 3 \times 1750}{60} = 550$ ft./sec.
5. $\left(\dfrac{2\pi rN}{60}\right)^2 = 550^2 = 302,500.$
6. $V' = 152.5\ (1 + \frac{1}{2}\ S) = 152.5 \times 1.064 = 162.4$ ft./sec.
7. $(V_r)^2 = (V')^2 + \left(\dfrac{2\ \pi\ rN}{60}\right)^2 = 26,370 + 302,500 = 328,900.$
8. $b\ (\text{ft.}) = \dfrac{7.65}{12} = .637$ ft.
9. $C_L = .510$ (from design data).
10. $L \text{ per ft.} = \dfrac{\rho\ C_L\ b\ (V_r)^2}{2} = \dfrac{.076 \times .510}{32.2 \times 2} \times .637 \times 328,900 = 136.5$ lb.
11. $S_t = \dfrac{M y_t}{I} = \dfrac{2350\ (\text{from graph})}{.283} = 8300$ lb./sq.in.

The total stress for the section is the sum of the centrifugal stress and bending stress, or,

$$\text{Total } S_t = S_t \text{ (cent)} + S_t \text{ (bend)} = 4180 + 8300 = 12,440 \text{ lb./sq.in.}$$

When the fact that the blades distort to some extent in practice is added to the original complex nature of the stresses, it will readily be seen that no great degree of accuracy can be expected from this analysis. It does, however, give comparative figures. If a propeller which is known to have sufficient strength for safe operation is analyzed for stresses, and the calculated stresses of similar propellers do not exceed the maximum calculated value of stress for the proven propeller, they may also be considered safe for trial flights. Since the stresses in geometrically similar propellers operating at the same V/nD vary in a certain definite manner, i.e., as the square of the tip speed, it is only necessary to keep below a predetermined tip speed to keep the stresses below a certain value. When the AR and CR are varied, however, different values of the tip speed are required to produce the same stress.

An Equation for Determining Relative Strength.

It was known from experience that a 10-foot oak propeller of low pitch having an AR of 6 and a CR of 1, could be safely operated on an airplane which had an R.P.M. of 1750 at maximum horizontal speed. Since the stresses are lower for propellers of higher pitch, all other factors remaining constant, the above case was taken as a conservative standard by which to judge the strength of other propellers. The stress was calculated for the case of the 10-foot propeller of AR = 6 and CR = 1 by means of the system given in this report. Assuming that the propeller turned 1750 R.P.M. at the maximum horizontal speed of the airplane, the highest stress found was 3268 lb./sq.in.

To find a relation between CR, AR, and tip speed (which may be taken as a constant times ND) for a constant value of stress, calculations were made for several standard propellers having different values of AR and CR. The value of ND was then found at which the stress of 3268 lb./sq.in. was obtained in each case. The results were plotted as shown in Figs. 4 and 5. Considering the curves as straight lines the following very simple equation can be derived:

$$CR = .0002\ ND + .3\ AR - 4.3.$$

This is called the "Constant Stress Formula" for if the CR is of the correct value to just equal the right-hand side of the equation, the maximum stress will be constant for all standard Navy oak propellers. If the CR is larger than necessary, the stresses will be lower; but if the CR is smaller than that indicated as necessary by the formula, the stresses will be higher than the safe standard.

It sometimes happens that with a particular combination of engine, airplane, and propeller, vibrations occur which may cause the propeller to fail even though calculations indicate that the strength is sufficient. This can usually be foretold in a wooden propeller by excessive flutter.

Conclusions

Owing to the fact that even the most complete and painstaking propeller stress analyses are inaccurate because of distortion and non-uniformity of material in wooden propellers, a simple relation such as the one described in this note is considered a satisfactory strength check for propellers of similar form. Its simplicity and ease of application make it a useful tool in the hands of a propeller designer.

Table I.

Stress Calculations
for 10 ft. Duralumin Propeller.

M.P.H. = 104   R.P.M. = 1750   S = .128

Centrifugal.

$$\frac{d(C.F.)}{dr} = Ar\ \frac{W}{g} \times \left(\frac{2\pi N}{60}\right)^2 = Ar \times \frac{.103}{32.2} \times \left(\frac{2\pi \times 1750}{60}\right)^2 = 108\ Ar.$$

Radius, in.	54	48	42	36	30	24	18	12
Radius, ft.	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0
b, inches	3.33	5.23	6.70	7.65	8.02	8.02	7.78	6.75
h $\frac{\text{upper}}{\text{lower}}$	.23	.35	.45	.57	.68	.84	$\frac{1.01}{.08}$	$\frac{1.19}{.79}$
Area, sq.in.	.567	1.35	2.23	3.23	4.04	4.99	6.27	9.89
A r, in.^{3}	30.6	64.8	93.6	116.4	121.2	119.7	113.0	118.6
$\frac{d(C.F.)}{dr}$	3305	7000	10110	12580	13100	12930	12200	12800
C.F. lb.	840	3460	7640	13380	19650	26000	32300	38800
$S_t = \frac{C.F.}{A}$ lb./sq.in.	1482	2560	3425	4140	4860	5210	5150	3920

Table I (Cont.)

Stress Calculations
for 10 ft. Duralumin Propeller.

M.P.H. = 104 R.P.M. = 1750 S = .128

Bending

V' = 1.064 x 1.467 x V = 162.4 ft./sec.

y_t	.096	.146	.187	.237	.283	.350	.470	.960
I	.00192	.0106	.0288	.0670	.1190	.225	.455	2.101
I/y_t	.020	.073	.154	.283	.420	.643	9.68	2.19
$\frac{2\pi rN}{60}$ ft/sec.	825	733	642	550	458	367	275	183
$\left(\frac{2\pi rN}{60}\right)^2$	680600	537300	412200	302500	209800	134700	75620	33490
$(V_r)^2$	707000	563700	438600	328900	236200	161100	92020	59890
b, ft.	.278	.436	.558	.637	.668	.668	.648	.594
C_L abs.	.450	.470	.490	.510	.570	.630	.760	.840
L, lb./ft.	104.7	136.Q	142.2	126.5	106.5	80.4	53.7	35.4
Shear, lb.	31.0	92.6	161.6	224.0	281.4	330.6	365.6	390.0
M, in.lb.	96	480	1170	2350	3820	5690	7760	10090
$S_t = \frac{My_t}{I}$ lb./sq.in.	4800	6580	7600	8300	9100	8850	8020	4600

Total Stress

S_t (cent.)	1482	2560	3425	4140	4860	5210	5150	3920
S_t (bend.)	4800	6580	7600	8300	9100	8850	8020	4600
S_t (total)	6282	9140	11025	12440	13960	14060	13170	8520

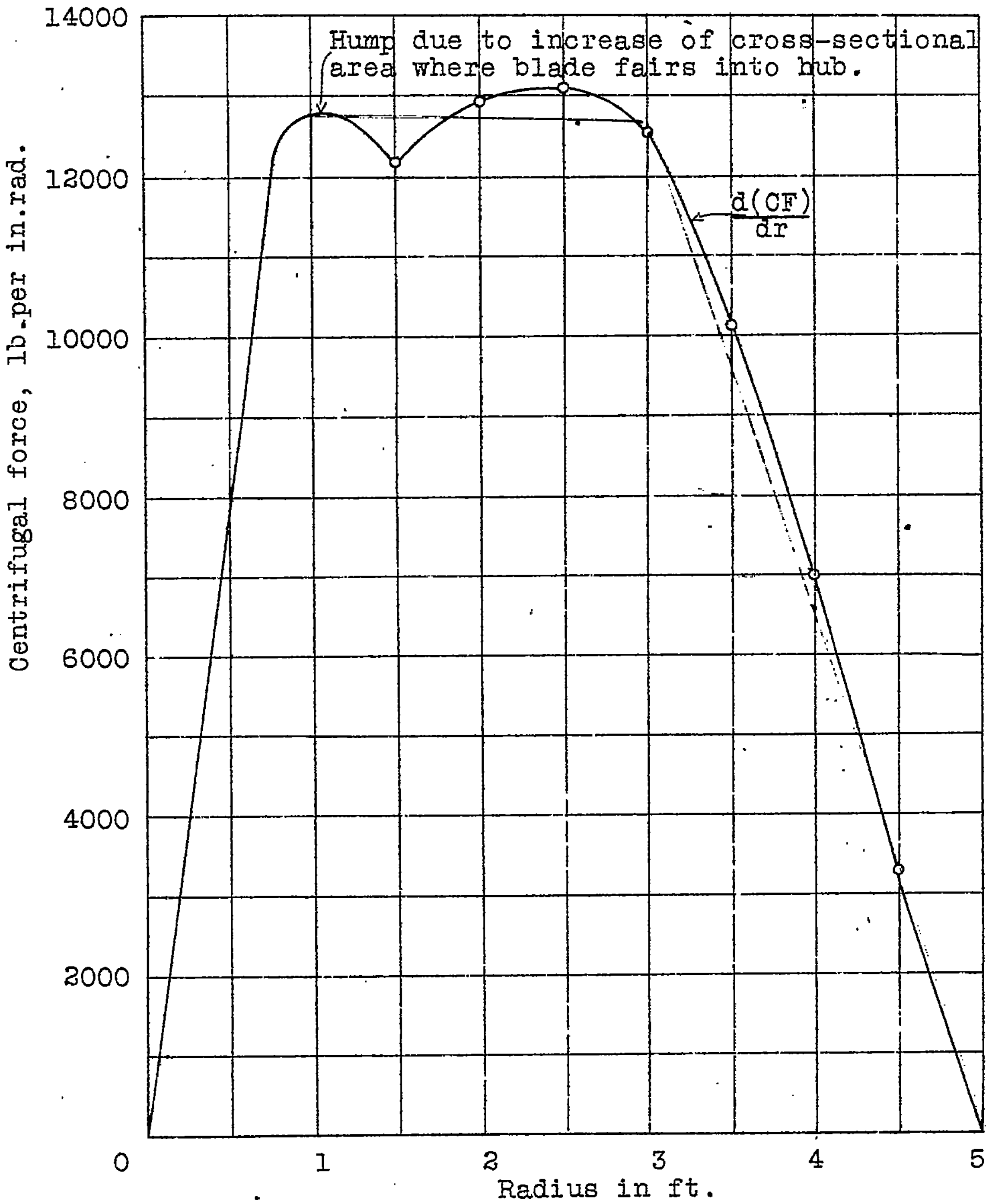

14000
12000
10000
8000
6000
4000
2000
0
1
2
3
4
5
Centrifugal force, lb.per in.rad.
Radius in ft.
Hump due to increase of cross-sectional area where blade fairs into hub.
d(CF)/dr

Fig.1

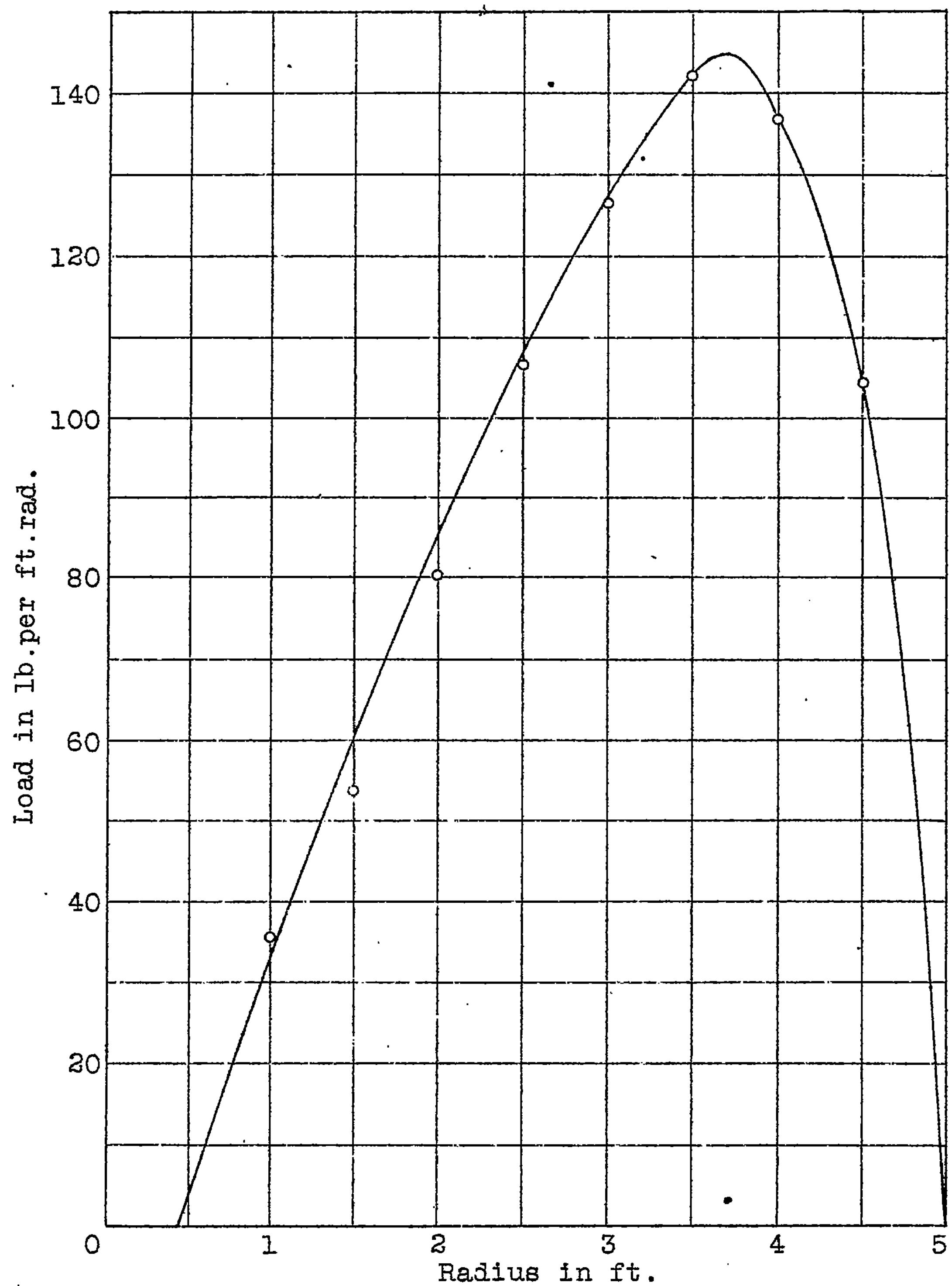


Fig.2 Load curve; area = shear; 1 sq.in.= 20 lb.

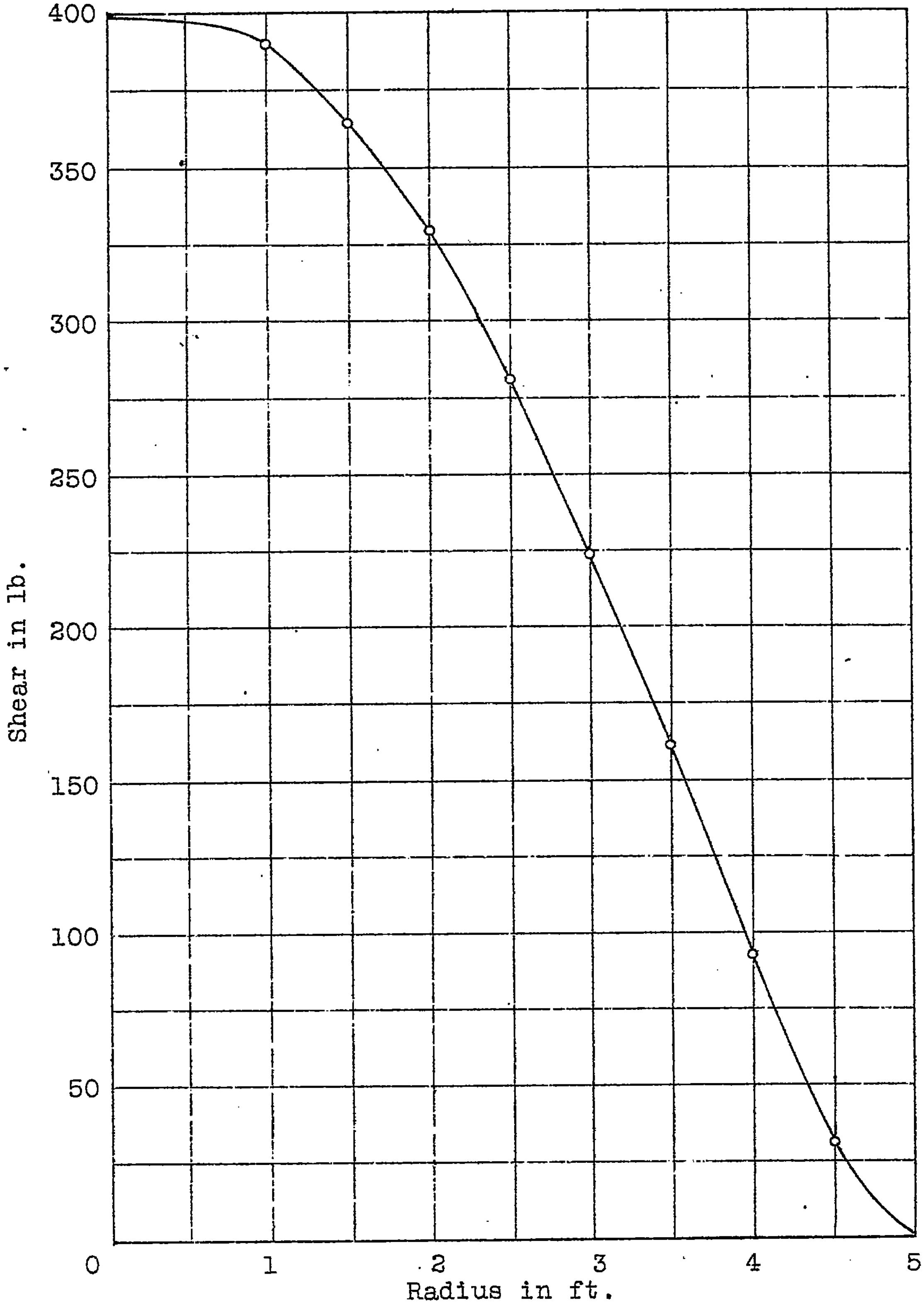


Fig.3 Shear curve; area = bending moment; 1 sq.in.= 50x12 = 600 in.lb.

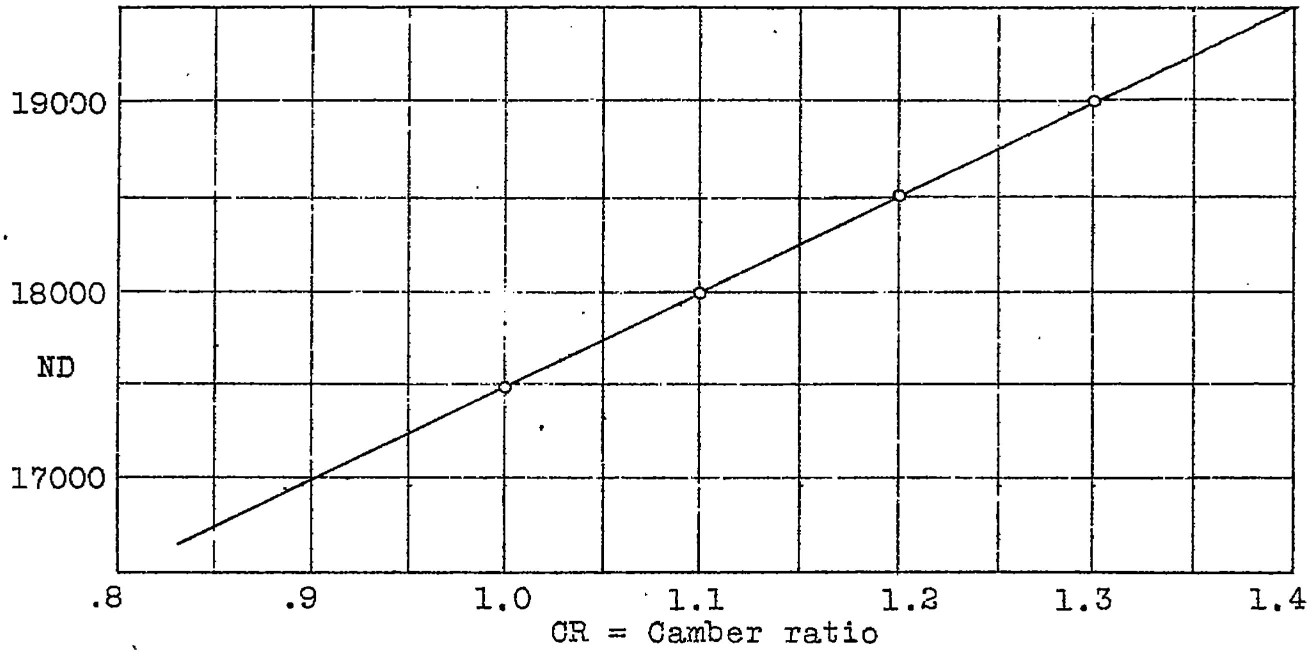


Fig.4 Variation of CR with ND for constant stress. AR = 6

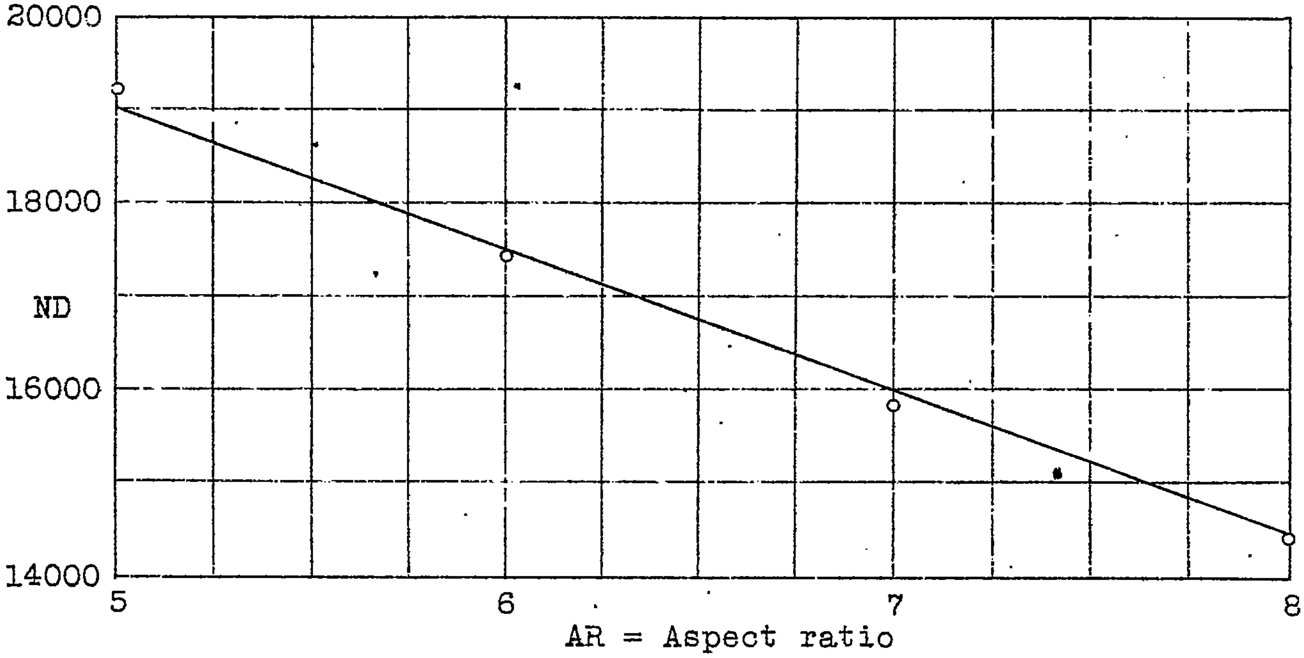
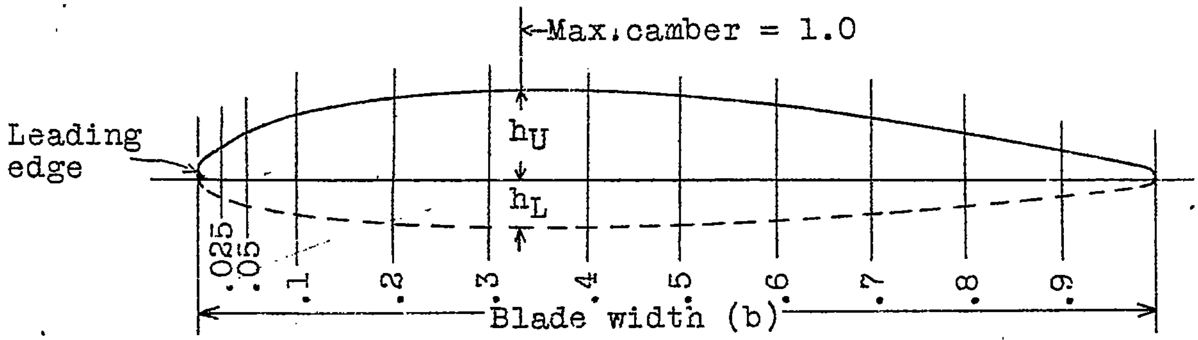
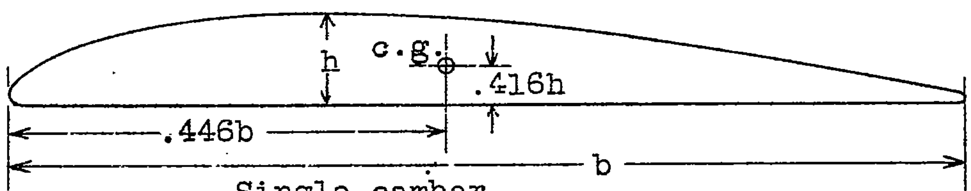


Fig.5 Variation of AR with ND for constant stress. CR = 1

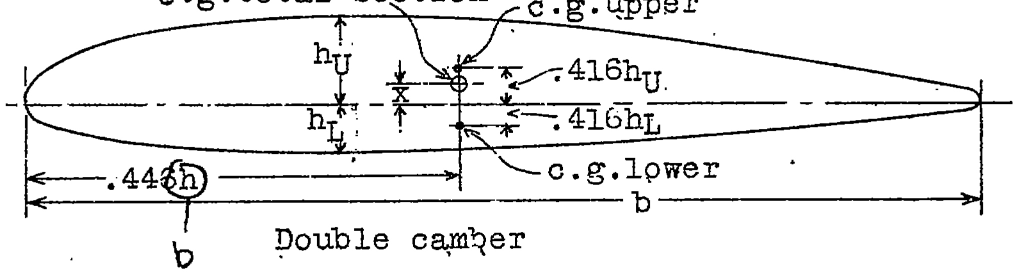


Station	LER	.025	.05	.1	.2	.3	.4	.5	.6	.7	.8	.9	TER
Ordinate	.10	.41	.59	.79	.95	.998	.99	.95	.87	.74	.56	.35	.077

Navy standard blade section. R.A.F.No.6 modified, flat face.



Single camber



Double camber

Area of upper section = .74 bh_U
" " lower " = .74 bh_L

$$x = \frac{.416h_U \times .74bh_U - .416h_L \times .74bh_L}{.74bh_U + .74bh_L} = .416(h_U - h_L)$$

Fig.6 c.g.and area of sections.